Goodness of Fit
Topic Test

1 H_0: The ratio 5:3:2 is a suitable model for the colours of female great Mormon butterflies

context

H_1: The ratio 5:3:2 is not a suitable model for the colours of female great Mormon butterflies

from ratio

show working at least once!

Butterfly colour	Observed	Probability	Expected	$\frac{(O-E)^2}{E}$
Black and Blue	82	0.5	66	$\frac{(82-66)^2}{66} = 3.879$ +
Black and Red	19	0.3	39.6	10.716 +
Pale White	31	0.2	26.4	0.802
Total	132	1	132	15.4

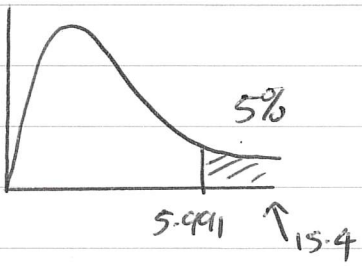
×132

Use a χ^2 Goodness of Fit test at the 5% level using $\nu = 3-1 = 2$

number of classes

Test statistic: 15.4

Critical region:



$15.4 > 5.991$

Result significant

Reject H_0

There is significant evidence to suggest the ratio 5:3:2 is not a suitable model for the colours of female Great Mormon butterflies.

context and not definite

from calculator.

2 (a) $\lambda = \bar{x} = 3.26$

Define variables → (b) Let X be the number of customer complaints.

$X \sim Po(3.26)$ per week.

0 3 [4] [5] ...

calculator → $P(X=4) = s = \underline{0.1807}$

$P(X \geq 5) = t = 1 - 0.0384 - 0.1251 - 0.204 - 0.2217 - 0.1807$

$= \underline{0.2302}$

show working at least once

x	Observed	Probability	Expected	$\frac{(O-E)^2}{E}$	
0	6	0.0384	9.60	$\frac{(6-9.60)^2}{9.60} = 1.35$	+
1	28	0.1251	31.29	0.346	+
2	46	0.2040	51.00	0.490	+
3	54	0.2217	55.42	0.0364	+
4	47	0.1807	45.18	0.0733	+
5+	69	0.2302	57.14	2.28	
Total	250	1	250	4.58	

×250

$u = 45.18$ $v = 57.14$

(c) H_0: The Poisson distribution is a suitable model for the number of customer complaints per week

H_1: The Poisson distribution is not a suitable model for the number of customer complaints per week

λ is estimated. Do not mention in hypotheses

context!

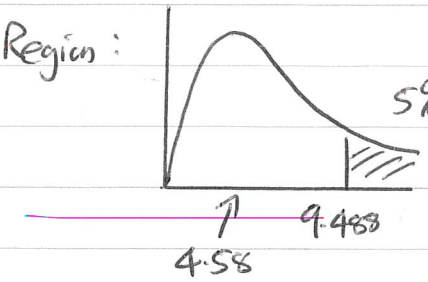
Use a χ^2 Goodness of Fit test at the 5% level using $\nu = 6 - 1 - 1 = 4$

number of classes; λ estimated

Test statistic: 4.58

2(c) (cont)

Critical Region:



$4.58 < 9.488$

Result not significant

Do not reject H_0

There is insufficient evidence to suggest the Poisson distribution is not a suitable model for the number of customer complaints per week.

No mention of λ / Context / Not definite

3 H_0: B(5, 0.02) is a suitable model for the number of patients who develop post-op. infections

H_1: B(5, 0.02) is not a suitable model for the number of patients who develop post-op. infections

n and p not estimated. Mention both!

Context

Let X be the number of patients who develop post-op infections

show working at least once

x	Observed	Probability	Expected	$\frac{(O-E)^2}{E}$
0	81	0.904	92.20	1.36
1	16	0.0922	9.41	+
2	4	3.76×10^{-3}	0.38	$\frac{(21-9.80)^2}{9.80} = 12.8$
3	1	7.68×10^{-5}	0.01	
4	0	7.84×10^{-7}	0	
5	0	3.2×10^{-9}	0	
Total	102	1	102	14.2

1-5 → 21 → 9.80

Must combine because some E's under 5

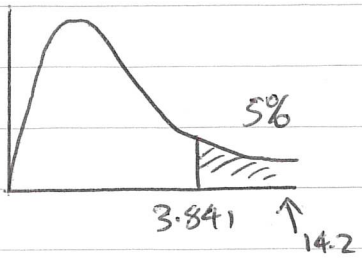
×102

Use a χ^2 Goodness of fit test at the 5% level using $\nu = 2 - 1 = 1$

Number of classes after combining

Test statistic: 14.2

Critical region:



$14.2 > 3.841$

Mention parameter / Context / Not definite

Result significant

Reject H_0

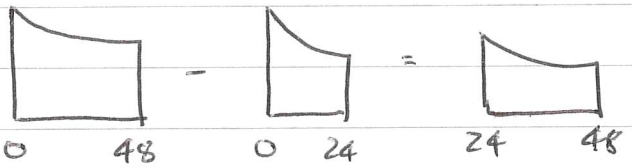
There is significant evidence to suggest $B(5, 0.02)$ is <u>not</u> a suitable model for the number of patients developing post-op infections

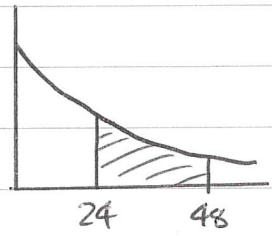
4 <u>a</u> Let X be the time between network outages.

Define variable ✓

$X \sim Exp(0.05)$ per hour

$P(24 \leq X \leq 48)$





Formula Book →

$$\left(1 - e^{-0.05 \times 48}\right) - \left(1 - e^{-0.05 \times 24}\right) = 0.210476\ldots$$

Expected Frequency $= 0.210476\ldots \times 50 = 10.524$ as required.

<u>b</u>

- There needs to be a "96+" category since the exponential distribution has no upper limit

- The classes 24-48, 48-72, 72-96 and 96+ need to be combined since the expected frequencies are under 5.

λ not estimated so mention it!

c H_0: Exp(0.05) is a suitable model for the time between network outages

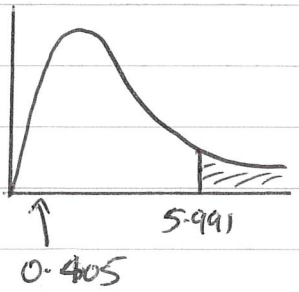
context → H_1: Exp(0.05) is not a suitable model for the time between network outages

Use a χ^2 Goodness of fit test at the 5% level using $\nu = 3 - 1$ (number of classes after combining)

$= 2$

Test statistic: 0.405

Critical region:



$0.405 < 5.991$

Result not significant

Do not reject H_0.

context not definite → There is insufficient evidence to suggest Exp(0.05) is <u>not</u> a suitable model for the time between network outages.

d Since there isn't enough evidence to suggest Exp(0.05) is <u>not</u> a suitable model for the time between network outages, $P_0(0.05)$ may be a suitable model for the number of network outages.

5 a) $\mu \approx \bar{x} = 303.65$
$\sigma \approx s_x = 43.07$
} From calculator

b) Use a formula to add up the values in cells B3 to F3 and multiply this by the value in cell B5.
e.g = sum(B3:F3)* B5

c) Cell F5: $1 - 0.129365 - 0.203182 - 0.271415 - 0.227541$
$= 0.168499$

Cell F6: $0.168499 \times 395 = 66.55$

Cell F7: $\frac{(86 - 66.55)^2}{66.55} = 5.68$

Cell G7: $5.68 + 25.22309 + \ldots + 0.03928 = 54.4$

μ and σ^2 estimated. Do not mention them

d) H_0: The normal distribution is a suitable model for the resolution of documents
H_1: The normal distribution is not a suitable model for the resolution of documents

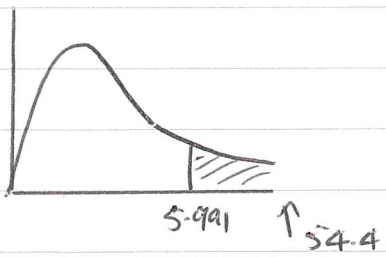
Context

Use a χ^2 Goodness of Fit test at the 5% level using $\nu = 5 - 2 - 1 = 2$

number of classes; μ and σ^2 estimated.

Test statistic: 54.4

Critical region:



$54.4 > 5.991$

No mention of μ or σ
Context
Not definite

Result significant.
Reject H_0. There is significant evidence to suggest the normal approximation is not suitable for the resolution of documents.